Semester 6					
Course Name	Ring Theory and Linear Algebra-II			**Total Credit**	5+1=6
Subject Course No.	MATH 61 HCC-XIII	**Core Course**	HCC-XIII	**Total Marks**	60+10+5=75

RING THEORY AND LINEAR ALGEBRA-II

Unit 1

Irreducible and prime elements, divisibility in integral domains, Euclidean domains, principal ideal domains, unique factorization domains and their relations. Greatest common divisor and least common multiple.

Polynomial rings over commutative rings, division algorithm and consequences, factorization of polynomials, irreducibility tests, Eisenstein criterion and unique factorization in $\mathbb{Z}[x]$.

Unit 2

Dual spaces, dual basis, double dual, transpose of a linear transformation and its matrix in the dual basis, annihilators. Eigen spaces of a linear operator, diagonalizability, invariant subspaces and Cayley-Hamilton theorem, the minimal polynomial for a linear operator, canonical forms.

Unit 3

Inner product spaces and norms, Gram-Schmidt orthogonalisation process, orthogonal complements, Bessel's inequality, the adjoint of a linear operator. Least squares approximation, minimal solutions to systems of linear equations. Normal and self-adjoint operators. Orthogonal projections and Spectral theorem.

Reference Books

- J. B. Fraleigh, A First Course in Abstract Algebra, 7th Ed., Pearson, 2002.
- M. Artin, Abstract Algebra, 2nd Ed., Pearson, 2011.
- J. A. Gallian, Contemporary Abstract Algebra, 4th Ed., Narosa Publishing House, 1999.
- S. H. Friedberg, A. J. Insel, Lawrence E. Spence, Linear Algebra, 4th Ed., Prentice- Hall of India Pvt. Ltd., New Delhi, 2004.
- S. Lang, Introduction to Linear Algebra, 2nd Ed., Springer, 2005.
- G. Strang, Linear Algebra and its Applications, Thomson, 2007.
- S. Kumaresan, Linear Algebra- A Geometric Approach, Prentice Hall of India, 1999.
- K. Hoffman, R. A. Kunze, Linear Algebra, Prentice-Hall of India Pvt. Ltd., 1971.
- S.H. Friedberg, A.L. Insel and L.E. Spence, Linear Algebra, Prentice Hall of India Pvt. Ltd., 2004.

Semester 6					
Course Name	Partial Differential Equations and Applications			**Total Credit**	5+1=6
Subject Course No.	MATH 61 HCC-XIV	**Core Course**	HCC-XIV	**Total Marks**	60+10+5=75

PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS

Unit 1

Partial differential equations – Basic concepts and definitions. Mathematical problems. First- order equations: classification, construction and geometrical interpretation. Method of characteristics for obtaining general solution of quasi linear equations. Canonical forms of first order linear equations. Method of separation of variables for solving first order partial differential equations.

Unit 2

Derivation of heat equation, wave equation and Laplace equation. Classification of second order linear equations as hyperbolic, parabolic or elliptic. Reduction of second order linear equations to canonical forms.

Unit 3

The Cauchy problem, Cauchy-Kowalewskaya theorem, Cauchy problem of an infinite string. Initial boundary value problems. Semi-infinite string with a fixed end, semi-infinite string with a free end. Equations with non-homogeneous boundary conditions. Non-homogeneous wave equation. Method of separation of variables, solving the vibrating string problem. Solving the heat conduction problem

Unit 4

Central force. Constrained motion, varying mass, tangent and normal components of acceleration, modelling ballistics and planetary motion, Kepler's second law.

Reference Books

- T. Myint-U and L. Debnath, Linear Partial Differential Equations for Scientists and Engineers, 4th edition, Springer, Indian reprint, 2006.
- S.L. Ross, Differential equations, 3rd Ed., John Wiley and Sons, India, 2004.
- M. L Abell, James P Braselton, Differential equations with MATHEMATICA, 3rd Ed., Elsevier Academic Press, 2004.
- I. N. Sneddon, Elements of Partial Differential Equations, McGraw Hill.
- F. H. Miller, Partial Differential Equations, John Wiley and Sons.
- S. L. Loney, An Elementary Treatise on the Dynamics of particle and of Rigid Bodies, Loney Press.

Semester 6					
Course Name	Linear Programming			**Total Credit**	5+1=6
Subject Course No.	MATH 62 DSE-III	**Discipline Specific Electives**	DSE-III	**Total Marks**	60+10+5=75

LINEAR PROGRAMMING

Unit 1

Introduction to linear programming problem (LPP), Problem formation, Type of solutions: Basic solution (BS), feasible solution (FS), basic feasible solution (BFS), degenerate and non-degenerate BFS. Matrix notation of LPP, graphical solution of LPP.

Unit 2

Theory of simplex method, convex sets, optimality and unboundedness, the simplex algorithm, simplex method in tableau format, introduction to artificial variables. Two-phase method, Big-M method and their comparison.

Duality, formulation of the dual problem, primal-dual relationships, economic interpretation of the dual.

Unit 4

Transportation and assignment problems: Mathematical formulation. North-west corner method, Least cost method and Vogel approximation method for determination of solution. Algorithm for solving transportation problem. Hungarian method for solving assignment problem.

Unit 4

Game theory: Formulation of two-person zero sum games, solving two-person zero sum games, games with mixed strategies, graphical solution procedure, linear programming solution of games.

Reference Books

- M. S. Bazaraa, J. J. Jarvis and H. D. Sherali, Linear Programming and Network Flows, 2nd Ed., John Wiley and Sons, India, 2004.
- F.S. Hillier and G.J. Lieberman, Introduction to Operations Research, 9th Ed., Tata McGraw Hill, Singapore, 2009.
- H. A. Taha, Operations Research, An Introduction, 8th Ed., Prentice-Hall India, 2006.
- G. Hadley, Linear Programming, Narosa Publishing House, New Delhi, 2002.

OR

Semester 6					
Course Name	Point Set Topology			**Total Credit**	5+1=6
Subject Course No.	MATH 62 DSE-III	**Discipline Specific Electives**	DSE-III	**Total Marks**	60+10+5=75

POINT SET TOPOLOGY

Unit 1

Countable and Uncountable Sets, Schroeder-Bernstein Theorem, Cantor's Theorem. Cardinal numbers and cardinal arithmetic. Continuum Hypothesis, Zorns Lemma, Axiom of Choice. Well-ordered sets, Hausdorff's maximal principle.

Unit 2

Topological spaces, basis and Sub basis for a topology, subspace topology, interior points, limit points, derived set, boundary of a set, closed sets, closure and interior of a set. Continuous functions, open maps, closed maps and homeomorphisms. Product topology, metric topology, Baire category theorem.

Unit 3

Connectedness. Distinguishing topological spaces via connectedness, intermediate value theorem, path connectedness, compact spaces, compact subspaces of the real line, limit point compactness.

Reference Books

- J. R. Munkres, Topology: A First Course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.
- J. Dugundji, Topology, Allyn and Bacon, 1966.
- G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw Hill, 1963.
- J. L. Kelley, General Topology, Van Nostrand Reinhold Co., New York,1995.
- J. Hocking, G. Young, Topology, Addison-Wesley Reading, 1961.
- L. Steen, J. Seebach, Counter Examples in Topology, Holt, Reinhart and Winston, New York, 1970.
- Adams and Franzosa, Introduction to topology, Pearson 2008.

Semester 6					
Course Name	Mathematical Modelling			**Total Credit**	5+1=6
Subject Course No.	MATH 62 DSE-IV	**Discipline Specific Electives**	DSE-IV	**Total Marks**	60+10+5=75

MATHEMATICAL MODELLING

Unit 1

Functions, modelling with linear and exponential functions. Average rate of change, linear functions with applications, Piecewise-linear functions with applications. Fitting linear models to data. Exponential growth functions with applications, Growth factors and rates, doubling time. Compound interest, Exponential decay functions with applications. Fitting exponential models to data, Decay factors and rates, Half-life. Modeling with logarithmic and polynomial functions, Logarithmic functions with applications, Fitting logarithmic models to data, Maxima and minima applications.

Unit 2

Introduction to continuous time models, limitations & advantages of the discrete-time model, the need for continuous time models, Continuous time models: the model for the growth of microorganisms, chemostat; Stability and linearization methods for system of ODE's.

Unit 3

Power series solution of Bessel's equation and Legendre's equation, Laplace transform and inverse transform, application to initial value problem up to second order.

Unit 4

Monte Carlo simulation modelling: simulating deterministic behavior (area under a curve, volume under a surface), generating random numbers: middle square method, queuing models. Overview of optimization modelling.

References Books

- T. Myint and L. Debnath, Linear Partial Differential Equation for Scientists and Engineers, Springer, Indian reprint, 2008.
- J. N. Kapoor, Mathematical Modelling, New Age International Pvt Ltd Publishers, 2011.
- K. Kamalanand and P. M. Jawahar, Mathematical Modelling of Systems and Analysis, PHI Learning Pvt Ltd, 2018.
- F. R. Giordano, W. P. Fox, S. B. Horton, A First Course in Mathematical Modeling, Brooks/Cole Cengage Learning, USA, 2013.

OR

Semester 6					
Course Name	Boolean Algebra and Automata Theory			**Total Credit**	5+1=6
Subject Course No.	MATH 62 DSE-IV	**Discipline Specific Electives**	DSE-IV	**Total Marks**	60+10+5=75

BOOLEAN ALGEBRA AND AUTOMATA THEORY

Unit 1 : Boolean Algebra

Lattice: Definition, examples and basic properties of ordered sets, maps between ordered sets, duality principle, lattices as ordered sets, lattices as algebraic structures, sublattices, products and homomorphisms. Definition, examples and properties of modular and distributive lattices.

Unit 2

Boolean algebra, Boolean polynomials, minimal and maximal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams. Logic gates, switching circuits and applications of switching circuits.

Unit 3 : Automata Theory

Introduction: Alphabets, strings and languages. Finite automata and regular languages: deterministic and non-deterministic finite automata, regular expressions, regular languages and their relationship with finite automata, pumping lemma and closure properties of regular languages.

Unit 4

Context free grammars and pushdown automata: Context free grammars (CFG), parse trees, ambiguities in grammars and languages, pushdown automaton (PDA) and the language accepted by PDA, deterministic PDA, Non deterministic PDA, properties of context free languages, normal forms, pumping lemma, closure properties, decision properties.

Unit 5

Turing Machines: Turing machine as a model of computation, programming with a Turing machine, variants of Turing machine and their equivalence.

References Books

- B. A. Davey and H. A. Priestley, Introduction to Lattices and Order, Cambridge University Press, Cambridge 1990.
- E. G. Goodaire and M. M. Parmenter, Discrete Mathematics with Graph Theory, (2nd Ed.), Pearson Education (Singapore) P. Ltd., Indian Reprint 2003.
- R. Lidl and G. Pilz, Applied Abstract Algebra, 2nd Edition, Undergraduate Texts in Mathematics, Springer (SIE), Indian reprint, 2004.
- J. E. Hopcroft, R. Motwani and J. D. Ullman, Introduction to Automata Theory, Languages and Computation, 2nd Ed., Addison-Wesley, 2001.
- H. R. Lewis, C. H. Papadimitriou, C. Papadimitriou, Elements of the Theory of Computation, 2nd Ed., Prentice-Hall, NJ, 1997.
- J. A. Anderson, Automata Theory with Modern Applications, Cambridge University Press, 2006.

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